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Projective geometry of systems of second-order differential equations

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Abstract

It is proved that every projective connection on an n-dimensional manifold M is locally defined by a system script capital L sign of n - 1 second-order ordinary differential equations resolved with respect to the second derivatives and with right-hand sides cubic in the first derivatives, and that every differential system script capital L sign defines a projective connection on M. The notion of equivalent differential systems is introduced and necessary and sufficient conditions are found for a system y to be reducible by a change of variables to a system whose integral curves are straight lines. It is proved that the symmetry group of a differential system script capital L sign is a group of projective transformations in n-dimensional space with the associated projective connection and has dimension ≤ n 2 + 2n. Necessary and sufficient conditions are found for a system to admit the maximal symmetry group; basis vector fields and structure equations of the maximal symmetry Lie algebra are produced. As an application a classification is given of the systems script capital L sign of two second-order differential equations admitting three-dimensional soluble symmetry groups. © 2006 RAS(DoM) and LMS.

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